Class XII

Mathematics

Set-3

Time: 3 hrs **M.M: 100 Marks**

General Instructions:

(i) All questions are compulsory.
(ii) Please check that this Question Paper contains **26** Questions.
(iii) Marks for each question are indicated against it.
(iv) Questions **1** to **6** in Section-**A** are Very Short Answer Type Questions carrying **one** mark each.
(v) Questions **7** to **19** in Section-**B** are Long Answer **I** Type Questions carrying **4** marks each.
(vi) Questions **20** to **26** in Section-**C** are Long Answer **II** Type Questions carrying **6** marks each.
(vii) Please write down the serial number of the Question before attempting it.

Section A

Q1 The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represent the two sides AB and AC, respectively of a. Find the length of the median through A.

Q2 Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i}-3\hat{j}+6\hat{k}$

Q.3 Find the maximum value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$$

Q4 If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

Q5 Matrix $A=\begin{vmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{vmatrix}$ is given to be symmetric, find values of a and b.

Q.6 Find the position vector of a point which divides the join of points with position vectors $\vec{a}-2\vec{b}$ *and* $2\vec{a}+\vec{b}$ externally in the ratio 2 : 1.

Section B

Q7 Find the general solution of the following differential equation :

$$\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$$

Q8 Show that the vectors $\vec{a}, \ \vec{b}$ *and* $\vec{c}$ are coplanar if $\vec{a}, \ \vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

Q9 Find the vector and Cartesian equations of the line through the point (1, 2, −4) and perpendicular to the two lines.

$\vec{r}=\left(8\hat{i}-19\hat{j}+10\hat{k}\right)+\lambda\left(3\hat{i}-16\hat{j}+7\hat{k}\right)$ and

$\vec{r}=\left(15\hat{i}+29\hat{j}+5\hat{k}\right)+\mu\left(3\hat{i}+8\hat{j}-5\hat{k}\right)$.

Q10 Three persons A, B and C apply for a job of Manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1 : 2 :4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.

OR

A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins.

Q11 Prove that: $tan^{-1}\frac{1}{5}+tan^{-7}\frac{1}{7}+tan^{-3}\frac{1}{3}+tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

OR

Solve for x: $2tan^{-1}\left(cos\,x\right)=tan^{-1}\left(2cosec\,x\right)$

Q12 The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves Rs 15,000 per month, find their monthly incomes using matrix method. This problem reflects which value?

Q13 If x = a sin 2t (1 + cos 2t) and y = b cos 2t (1 – cos 2t), find the values of $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ and $t=\frac{\pi}{3}$.

OR

If $y = X^x$, *prove that* $\frac{d^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=0.$

Q14 Find the values of p and q for which

$$f(x)=\begin{cases}\frac{1-\sin^3 x}{3\cos^2 x}, if\ x<\frac{x}{2}\\ P\quad, if\ x=\frac{\pi}{2}\\ \frac{q(1-\sin x)}{(\pi-2x)^2}\ , if\ x>\frac{\pi}{2}\end{cases}$$ is continuous at $x=\frac{\pi}{2}$

Q15. Show that the equation of normal at any point t on the curve $x = 3\cos t - \cos^3 t$ and $y = 3\sin t - \sin^3 t$ is $4\left(y\cos^3 t - \sin^3 t\right) = 3\sin 4t.$

Q16. Find $\int\frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta}\theta.$

OR

Evaluate $\int_0^{\pi} e^{2x}.\ \sin\left(\frac{\pi}{4}+x\right)dx$

Q17. Find $\int\frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx.$

Q18 Evaluate $^2\int_{-1}|x^3-x|dx.$

Q19 Find the particular solution of the differential equation $\left(1-y^2\right)(1+logx)dx+2xydy=0$ given that y = 0 when x = 1.

SECTION C

Q20 Find the coordinate of the point P where the line through A(3, –4, –5) and B(2, –3, 1) crosses the plane passing through three points L(2, 2, 1), M(3, 0, 1) and N(4, –1, 0).
Also, find the ratio in which P divides the line segment AB.

Q.21 An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also find mean and variance of the distribution.

Q22 A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at Rs 7 profit and B at a profit of Rs 4. Find the production level per day for maximum profit graphically.

Q23 Let $f : N \to N$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f : N \to S$, where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ *and* $f^{-1}(163)$.

Q24 Prove that $\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$ is divisible by (x + y + z) and hence find the quotient.

OR

Using elementary transformations, find the inverse of the matrix

$A = \begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ and use it to solve the following system of linear equations :

$$8x + 4y + 3z = 19$$
$$2x + y + z = 5$$
$$x + 2y + 2z = 7$$

Q.25 Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find maximum volume in terms of volume of the sphere.

OR

Find the intervals in which $f(x) = \sin 3x - \cos 3x,\ 0 < x < \pi,$ is strictly increasing or strictly decreasing.

Q26 Using integration find the area of the region $\{(x, y) : x^2 + y^2 \leqslant 2ax, y^2 \geqslant ax, x, y \geqslant 0\}$.